A Scheme for Multiuser Communications based on Energy Division

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Abstract—**We discuss a multiple-access protocol that discriminates different users on the basis of their energy. Users are assumed to share the same bandwidth, the same pulse and are decoded according to their received magnitudes. This proposal promises to achieve larger efficiency when jointly applied with classical protocols especially in scenarios where the same low-cost terminals are deployed in large numbers as in sensor networks and in personal-area networks. Analytical results, confirmed by numerical simulations, are derived for performance evaluation on additive white Gaussian noise channels.**

I. Introduction

Classical multiple access systems for communication are based on time, frequency or code division. Their advantages and disadvantages have been discussed in the classical literature [1], [2], [3], [4].

The increasing demand of bandwidth in modern applications requires to consider scenarios in which resources allocable with the classical schemes have been saturated. In such cases, several users may need to use simultaneously the same bandwidth with the same baseband pulse and still be able to get multiple access at the receiver. Moreover, in personal-area networks and sensor networks, the infrastructure availability is often not known *a priori* and simple access schemes are desirable.

This paper explores a multiple access scheme that allows several users to transmit simultaneously with the same baseband pulse $p(t)$ according to a binary phase shift keying (BPSK). The information bit transmitted by each user is recovered by the receiver exploiting the differences in the received magnitude from each user. Such a system, here called Energy-Division Multiple Access (EDMA), is simple to be implemented and manages multiple access via a slight generalization of the power control procedures [5]. From the receiver point of view, EDMA is equivalent to single pulse-amplitude-modulation (PAM) signaling, but appropriate constraints must be introduced to make the receiver problem solvable. The achieved bit error rate (BER) on additive white Gaussian channel (AWGN) is analytically evaluated and the obtained results are confirmed by computer simulations.

II. The Proposed Model

Consider a set of N users transmitting to a single receiver on a single AWGN channel using simultaneously the same pulse for a simple BPSK modulation. Although our approach can be applied to more sophisticated scenarios, we focus here, for sake of clarity, on the simplest case in which users transmissions are assumed synchronous at the receiver. The baseband discrete-time signal, after matched filtering and

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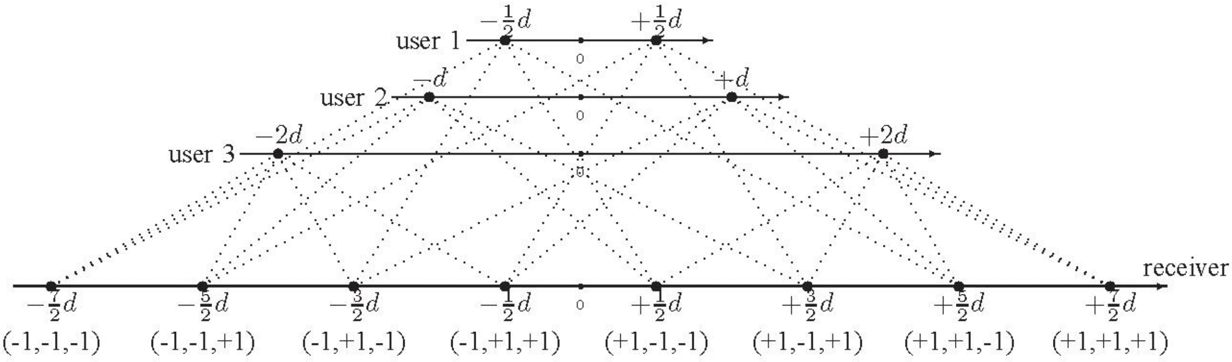

Fig. 1. Constellation in EDMA with 3 users.

sampling at the symbol rate, is written as

$$y = \sum_{n=1}^{N} a_n b_n + w = \mathbf{a}^{\mathrm{T}}\mathbf{b} + w = x + w \ ,$$

where $w \sim \mathcal{N}(0, \sigma^2)$ is the overall additive noise, $b_n \in \{-1, +1\}$ is the bit of the nth user, $\mathbf{b} = (b_1, \ldots, b_N)^{\mathrm{T}} \in \{-1, +1\}^N$ is the *transmission vector*, and $\mathbf{a} = (a_1, \ldots, a_N)^{\mathrm{T}}$ is the *level vector*, where each signal level a_n is expressed as

$$a_n = \frac{k_n}{r_n}\sqrt{\mathcal{E}_n} \ .$$

Level a_n is determined by $\mathcal{E}_n$, r_n, and k_n that denote the energy transmitted by the nth user, its distance from the receiver, and a positive constant depending on the specific scenario (propagation constant, antenna gain, etc.), respectively.

The problem of recovering $\mathbf{b}$ from the scalar y appears though, since we are transmitting at same time, on the same band and with same code. However, the contributions from different users can in fact be discriminated on the basis of their energy because if users are at different distances from the receiver, they should be naturally differentiable on the basis of their received magnitude. Furthermore if some cooperation is possible, users can adjust their power levels enhancing or creating such differences. The application scenario may be one in which users interact initially with the receiver on the basis of an ALOHA protocol, like in RFID standard [6], and once mutual arrangements are completed, start transmitting synchronously at full speed. Such an idea is only at an embryonic stage in ALOHA with capture [7].

The 2^N configurations of $\mathbf{b}$ constitute the 2^N-ary PAM constellation, $x \in \{s_1, \ldots, s_{2^N}\}$. Figure 1 shows the case for $N = 3$. The association of the scalar value y to a constellation point is feasible if there is a one-to-one mapping between x and $\mathbf{b}$. In other words no pair of bit configurations can correspond to the same constellation point. It can be easily shown that this imposes to the following *Separability Condition*

$$\sum_{n=1}^{N} v_n a_n \neq 0 \ ,$$

for each combination with $v_n \in \{-1, 0, +1\}$ except the one with all null coefficients. This is equivalent to say that each different pair $\mathbf{b}_i$ and $\mathbf{b}_j$ has to correspond to two different values s_i and s_j. Even if such a condition is satisfied, the constellation points may still be very confused on the observation axis, and the association of the constellation point to each user's bit may be somewhat cumbersome. For the receiver to remain simple and return directly the user's bits via threshold detection, user n can be threshold-detected if for $\sigma \to 0$

$$y = a_n b_n + \sum_{m=1, m \neq n}^{N} a_m b_m + w \quad \begin{cases} > 0 & \text{if } b_n = +1 \\ < 0 & \text{if } b_n = -1 \end{cases} \ ,$$

regardless of the bit configurations for the other users. By straightforward bounding, this requires $a_n > \sum_{m=1, m \neq n}^{N} a_m$. Assuming without loss of generality that users magnitudes are ranked as $0 < a_1 < \ldots < a_N$, a little thought reveals that a sufficient condition to satisfy separability for all users is

$$a_n > \sum_{m=1}^{n-1} a_m \qquad n = 2, \ldots, N \ .$$

Such a constraint allows conditional separability of each user on the decision taken on the previous ones: fix a threshold and decide on user N, than decide on user $(N-1)$ on the basis of the previous decision, etc. The procedure evolves until user 1 has been decoded (successive cancellations). This sufficient condition has been derived within an overloaded CDMA scenario [8] and also discussed in [9], and guarantees that as noise goes to zero the BER vanishes for all users.

Since asymptotic BER depends essentially on the minimum distance among constellation points, users can share similar asymptotic performances if the overall PAM constellation is

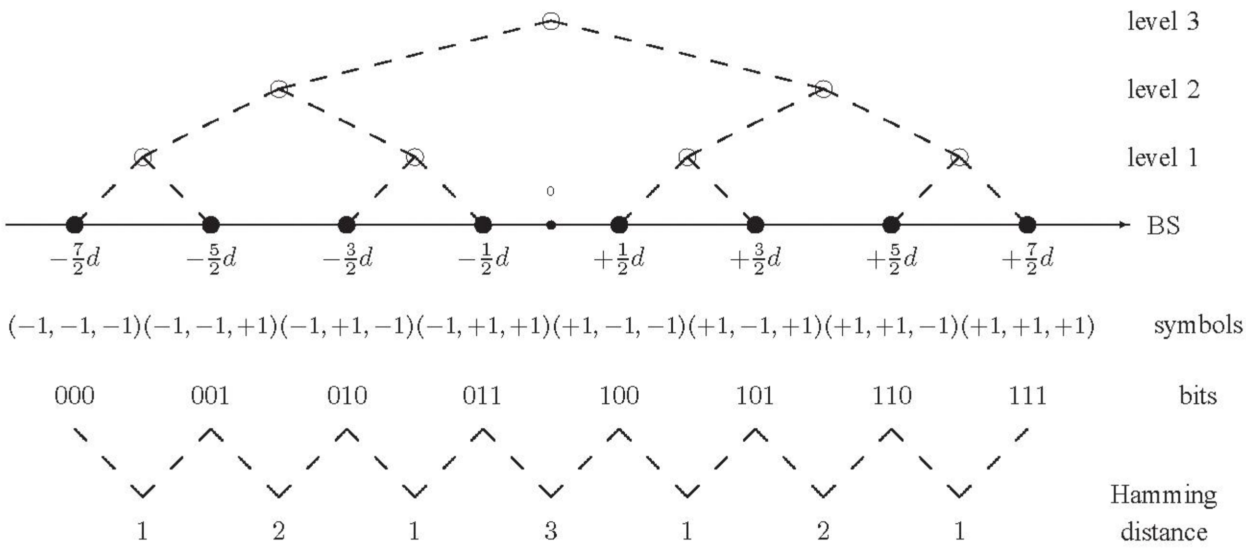

Fig. 2. Tree structure for a system with 3 users.

constrained to be uniform. This is obtained if we impose

$$a_n = \frac{d}{4} 2^n ,$$

where d denotes the distance between two adjacent symbols. Note that no Gray coding is allowed because the code is imposed by the linear mapping.

The average energy spent on each channel use is easily computed to be

$$\mathcal{E}_{\text{av}} = \frac{1}{N} \sum_{n=1}^{N} \mathcal{E}_n = \frac{2^{2N}-1}{12N} d^2 ,$$

while the average SNR is defined as follows

$$\Gamma_{\text{av}} = \frac{\mathcal{E}_{\text{av}}}{2\sigma^2} .$$

III. Performance Analysis

System performance can be evaluated in terms of joint error probability $P_{\text{joint}}(e)$ and average single-user error probability $P_{\text{av}}(e)$ [1]. The former accounts for the error rate on the equivalent symbol of the overall constellation and it can be expressed [3] as

$$P_{\text{joint}}(e) = \frac{2^N-1}{2^N} \operatorname{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\text{av}}}\right) ,$$

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \exp(-t^2) dt .$$

The latter accounts for the error rate on the user bit and it is computed as the ratio between the average number of bits in error over the number of transmitted bits. The code induced by the magnitude ordering $a_1 < \ldots < a_N$ is a simple BCD code with the farthest user corresponding to the least significant bit (see Figure 2). Assuming uniform *a priori* probabilities, $P_{\text{av}}(e)$ can be written as

$$P_{\text{av}}(e) = \frac{1}{N2^N} \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} p(j|i) e(i,j)$$

where $p(j|i)$, defined as the (i,j)th entry of the *channel matrix* $\mathbf{P}$, represents the probability that the jth symbol of the overall PAM is received when the ith symbol has been transmitted. Similarly $e(i,j)$, defined as the (i,j)th entry of the *bit-error matrix* $\mathbf{E}$, is the number of bits in error between bit configuration $\mathbf{b}_i$ and $\mathbf{b}_j$.

Under the assumption that the errors only occur between adjacent points on the overall PAM (large SNR approximation) the channel matrix $\mathbf{P}$ can be approximated with the $2^N \times 2^N$ tri-diagonal matrix

$$\mathbf{P} \approx \begin{pmatrix} q_1 & q_{\text{in}} & 0 & \ldots & 0 & 0 \\ q_{\text{out}} & q_0 & q_{\text{in}} & \ddots & \vdots & 0 \\ 0 & q_{\text{in}} & q_0 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & q_{\text{in}} & 0 \\ 0 & \ldots & 0 & q_{\text{in}} & q_0 & q_{\text{out}} \\ 0 & 0 & \ldots & 0 & q_{\text{in}} & q_1 \end{pmatrix}$$

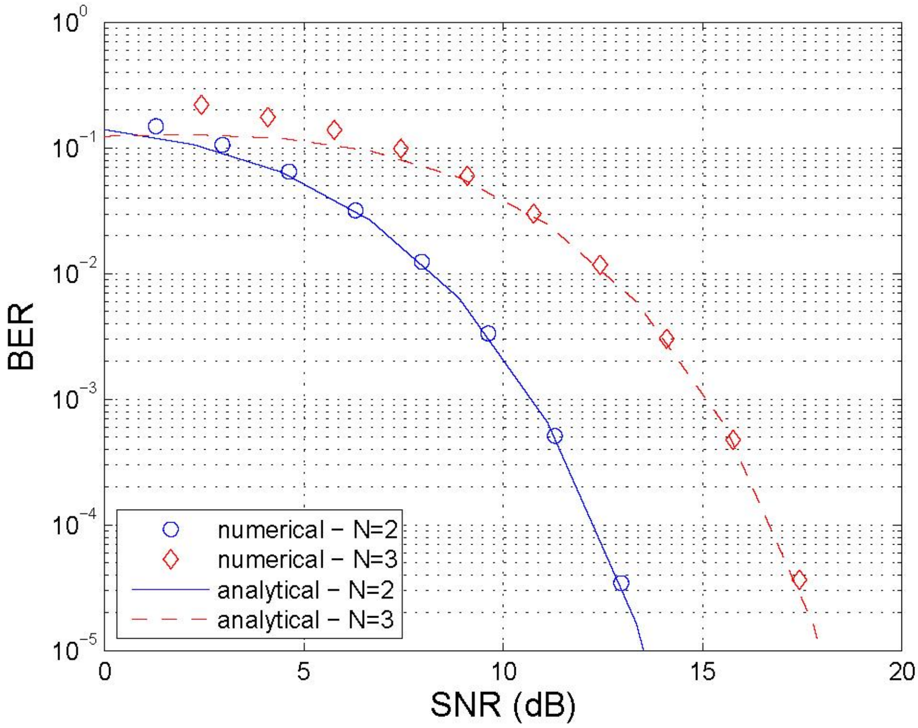

Fig. 3. Average performances of the single user obtained by numerical simulations and by analytical calculations.

where

$$\begin{aligned} q_0 &= 1 - \operatorname{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) \\ q_1 &= 1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) \\ q_{\mathrm{in}} &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{27N}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) \\ q_{\mathrm{out}} &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) . \end{aligned}$$

The corresponding bit-error matrix $\mathbf{E}$ is symmetric and has the main diagonal with null elements. The other two diagonals that enter into the computation of $P_{\mathrm{av}}(e)$ are equal and are described by the $(2^N - 1)$-dimensional column vector $\mathbf{c}(N)$. It is straightforward (refer to Figure 2) to show that $\mathbf{c}(n)$ can be computed inductively as

$$\begin{cases} \mathbf{c}(1) = 1 \\ \mathbf{c}(n) = \left(\mathbf{c}(n-1)^{\mathrm{T}}, n, \mathbf{c}(n-1)^{\mathrm{T}}\right)^{\mathrm{T}} \end{cases} .$$

For example $\mathbf{c}(2) = (1,2,1)^{\mathrm{T}}$; $\mathbf{c}(3) = (1,2,1,3,1,2,1)^{\mathrm{T}}$; $\mathbf{c}(4) = (1,2,1,3,1,2,1,4,1,2,1,3,1,2,1)^{\mathrm{T}}$. Finally denoting with $\mathbf{q}(N)$ a column vector whose first element is q_{out} and the remaining $2^N - 2$ are q_{in}, we obtain

$$P_{\mathrm{av}}(e) \approx \frac{2}{N 2^N}\mathbf{c}(N)^{\mathrm{T}}\mathbf{q}(N) .$$

System performance with 2 and 3 users have been verified by computer simulations. The analytical and numerical results (for up to 10^7 trials) are shown in Figure 3. To give an idea of the performance range among users we have also derived, along the same lines of [10], the error probability of the farthest and the nearest users[1] ($P_e(1)$ and $P_e(N)$), obtaining

$$\begin{aligned} P_e(1) &= \frac{1}{2^N}\sum_{n=1}^{2^N-1}(2^N - n)\operatorname{erfc}\left(\sqrt{\frac{3N(2n-1)^2}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) \\ P_e(N) &= \frac{1}{2^N}\sum_{n=1}^{2^{N-1}}\operatorname{erfc}\left(\sqrt{\frac{3N(2n-1)^2}{2^{2N}-1}\Gamma_{\mathrm{av}}}\right) . \end{aligned}$$

Analytical and numerical results, for systems with 2 and 3 users, are reported in Figure 4. Both Figures 3 and 4 show excellent agreement.

IV. Comments and Conclusion

Multiple access on the same band and with the same pulse is possible if users magnitudes are properly controlled. Our analyical results agree with computer simulations making EDMA a feasible idea. Various deployment scenarios are possible ranging between a configuration in which the users are all at the same distance from the receiver ($k_n/r_n = \text{const}$) and a configuration in which they are at such distances that the required separability condition ($k_n/r_n = \text{const } 2^n$) is satisfied without power control. None of the two is realistic and

[1]Farthest and nearest refer to the signal level, not to the physical distance.

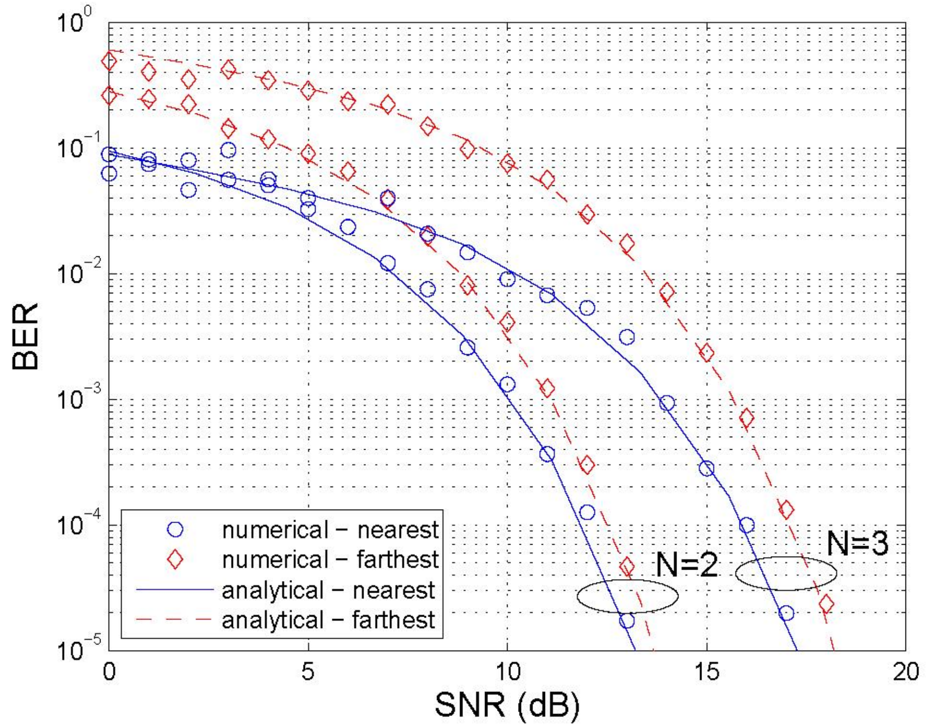

Fig. 4. Individual performances of the nearest and farthest users obtained by numerical simulations and by analytical calculations.

practical systems are clearly somewhere in between. However, focusing on the two extremes may be enlightening. In the first configuration, multiple access must be achieved by controlling the energy spent by each user, but different BERs would be experienced. Fairness (both in terms of energy and BER) could be achieved by periodically rotating energy assignments, as simple modifications of power control procedures [5]. In the second configuration, users can be managed without any further energy consumption (thus achieving fairness in terms of energy), even though the farthest user will experience the worst performance. Practical cases will clearly require compromises between fairness and energy optimization. More complex configurations could be also conceived by grouping users and receivers.

References

[1] S. Verdú, *Multiuser Detection*, Cambridge University Press, 1998.

[2] S. Haykin, *Digital Communications*, John Wiley and Sons, 2000.

[3] J.G. Proakis, *Digital Communications*, McGraw Hill, 2000.

[4] T.F. Rappaport, *Wireless Communications: Principles and Practice*, 2nd Edn., Prentice Hall, 2002.

[5] J. Zander, "Performance of Optimum Transmitter Power Control in Cellular Radiosystems," *IEEE Transactions on Vehicular Technology*, Vol. 41(1), pp. 57-62, February 1992.

[6] K. Finkenzeller, *RFID Handbook*, John Wiley and Sons, 1999.

[7] B. Ramamurthi, A.A.M. Saleh, D.J. Goodman, "Perfect-Capture ALOHA for Local Radio Communications," *IEEE Journal on Selected Areas in Communications*, Vol. 5(5), pp. 806-814, June 1987.

[8] G. Romano, F. Palmieri, P.K. Willett, D. Mattera, "Separability and Gain Control for overloaded CDMA," *Conference on Information Sciences and Systems*, March 2005.

[9] P. Salvo Rossi, G. Romano, D. Mattera, F. Palmieri, S. Marano, "An Energy-Division Multiple Access Scheme," *International Conference on Information Fusion*, July 2006.

[10] K. Cho, D. Yoon, "On the General BER Expression of One- and Two-Dimensional Amplitude Modulations," *IEEE Transactions on Communications*, Vol. 50(7), pp. 1074-1080, July 2002.